Tutorial Assignment 3: Dilution and Magnetic Refrigerators

Handwriting should be legible. Calculations should be explained. Units should be given. Numerical answers should be given to 4 significant figures.

1. Liquid ^{3}He may be modelled as Fermi gas, if we assume that the effective mass is about 2.5 times the actual mass of the atom. Consider two phases of ^{3}He: pure ^{3}He, and a 6.6% (in moles) solution in ^{4}He. Assume a molar volume of 30 cm^3 for both liquid ^{4}He and ^{3}He.

   i) Find the Fermi temperatures of the two phases. [2]

   ii) Write down the formula for the heat capacity for one mole of ^{3}He. Find them for both phases, in the form γT, where γ is a number you should determine. [2]

   iii) Using the relation betwen heat and entropy, $dQ = TdS$, find the entropies of the phases in terms of T. [2]

   iv) Then find the heat change when one mole of ^{3}He moves from the pure phase to the solution phase at 10 mK. Explain why there is a cooling effect. [2]

   v) When concentration of the solution decreases, does the cooling effect increase or decrease? Explain. [2]

2. A sample of the salt CMN containing one mole of spin ½ ions, sits in a field of 1 T.

   i) Find the energy difference between the magnetic levels. What are the populations of the levels at the high temperature limit? [2]

   ii) The temperature is lowered to 1 K. Find the ratio of the Boltzmann factors of the two levels. Hence find the new populations and the heat given out by the salt. [2]

   iii) Then the sample is thermally insulated from the surrounding, and the field lowered to 0.1 T. Explain how to find the new temperature, and find it. [2]

   iv) After some time, the temperature slowly rises, because the insulation is not perfect. Assuming that the populations become nearly equal, find the heat absorbed by the salt. What is the name for the heat absorbed? [2]

   v) Sketch an entropy versus temperature graph showing the path of the above cycle. Indicate the areas that show the heat given out and the heat absorbed. [2]

CONSTANTS

Speed of light in vacuum	c	=	3.00×10^{8} ms^{-1}
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$ Hm^{-1}
		=	$4\pi \times 10^{-7}$ VsA^{-1}m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12} Fm^{-1}
		=	8.85×10^{-12} AsV^{-1}m^{-1}
Elementary charge	e	=	1.60×10^{-19} C
Planck constant	h	=	6.63×10^{-34} Js
	$h/2\pi = \hbar$	=	1.05×10^{-34} Js
Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	=	1.38×10^{-23} JK^{-1}
Gas constant	R	=	8.31 JK^{-1}mol^{-1}
Unified atomic mass constant	m_u	=	1.66×10^{-27} kg
		=	931.5 MeVc^{-2}
Electron mass	m_e	=	9.11×10^{-31} kg
Proton mass	m_p	=	1.67×10^{-27} kg
Gravitational constant	G	=	6.67×10^{-11} Nm^{2}kg^{-2}
Acceleration due to gravity	g	=	9.81 ms^{-2}
Bohr magneton	μ_B	=	9.27×10^{-24} JT^{-1}